Cosmological Tests for a Linear Coasting Cosmology

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Abstract

A strictly linear evolution of the scale factor is a characteristic feature in several classes of alternative gravity theories. In this article we investigate the overall viability of an open linear coasting cosmological model. We report that this model is consistent with gravitational lensing statistics (within 1σ) and accomodates old high-redshift galaxies. We finally conclude that such a linear coasting, $\alpha(t) = t$, is not ruled out on basis of these observational tests.

1   Introduction

Standard cold dark matter (CDM) cosmology presents serious theoretical and observational problems as a model for an acceptable description of the Universe. This has motivated a search for alternative cosmological models [1]. The first problem comes from the conflict between age of the Universe and the age of the oldest stars in Galactic globular clusters. The age constraints from old galaxies at high redshifts render "the age problem" even more acute [2].

Another major difficulty is the cosmological constant (Λ) problem. While it is difficult to have a decent theoretical justification for Λ [3], there are serious doubts on the compatibility of the Hubble Deep Field (HDF) data with large Λ. Initial analysis of Maoz and Rix [4] placed a somewhat stringent upper limit of $\Omega_\Lambda < 0.7$ which have been somewhat softened by later analysis of Cooray et al. [5].

These problems have generated a lot of interest in an open FRW model with a linear evolution of the scale factor, $a(t) \propto t$. In such a cosmology the universe expands with constant velocity; hence the term coasting cosmology [6]. Notable among such models is a

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recent idea of Allen [7], in which such a scaling results in an $SU(2)$ cosmological instanton dominated universe. The Weyl gravity theory of Manheim and Kazanas [8] makes space for yet another possibility. Here again the FRW scale factor approaches a linear evolution at late times.

The need for investigating such a model comes from several considerations. Particle horizons occur in models with $a(t) \approx t^\alpha$ for $\alpha < 1$ and, therefore, linear coasting model does not suffer the horizon problem. Also, linear evolution of a scale factor is supported in alternative gravity theories (eg. non-minimally coupled scalar-tensor theories), where it turns out to be independent of the matter equation of state (see [9] and references therein). The scale factor in such theories does not constrain the matter density parameter and, therefore, does not present any flatness problem. Moreover, the age of the coasting universe is 1.5 times the age of standard CDM universe [9], which makes this model comfortably concordant with the ages of globular clusters. Finally, a linear coasting cosmology, independent of the equation of state of matter, is a generic feature in a class of models that attempt to dynamically solve the Λ problem [3]. Such models have a scalar field non-minimally coupled to the curvature of the universe. With the evolution of time, the non-minimal coupling diverges, the scale factor quickly approaches linearity and the non-minimally coupled field acquires a stress energy that cancels the vacuum energy in the theory.

Interestingly it was noted by Perlmutter et al. [10] that the curve for $\Omega_M = \Omega_\Lambda = 0$ (for which the scale factor would have linear evolution) is "practically identical to the best fit plot for an unconstrained cosmology". Recently, it was shown by Dev et al. [11] that open linear coasting cosmology presents a good fit to the SNe Ia data. It was also demonstrated that this model is consistent with the primordial nucleosynthesis [12].

In this paper we consider constraints on the index α of a power law cosmology, $a(t) \propto t^\alpha$, from two different tests: gravitational lensing statistics and age estimates of old high-redshift galaxies. The expected frequency of multiple imaging lensing events is a sensitive probe for the viability of a given cosmology. In view of the successful results of the above mentioned works [11, 12], we used this test to constrain the power index α of the scale factor. Expected number of lens systems depends upon the index α through the angular diameter distances. By varying α, the number of lenses changes which on comparison with the observations gives us the constrain on α. Age measurements of old high-redshift galaxies give lower bound on the power index α. This is based on the fact that the age of the universe in a given redshift is greater than or at least equal to the age of its objects. Since the age of the universe is a function of α, we find the value of α which permits the existence of these old galaxies.

In Section 2 we introduce the ansatz for the power law cosmology and derive the angular diameter distance formula. In Section 3 we describe the tests and the constraints they present on the power index for that cosmology. We summarize our results and present the result of a joint constraint from various observational tests in Section 4.

2 Linear Coasting Cosmology

We consider a general power law cosmology with the scale factor given in terms of two arbitrary dimensionless parameters B and α

$$a(t) = B\frac{c}{H_0}\left(\frac{t}{t_0}\right)^\alpha , \tag{1}$$

for an open FRW metric

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] . \tag{2}$$

Here t is cosmic proper time and r, θ, φ are comoving spherical coordinates.

The expansion rate of the universe is described by a Hubble parameter, $H(t) = \dot{a}/a = \alpha/t$. The *present* expansion rate of the universe is defined by a Hubble *constant*, equal in our model to $H_0 = \alpha/t_0$ (here and subsequently the subscript 0 on a parameter refers to its present value). The scale factor and the redshift are related to their present values by $a/a_0 = (t/t_0)^\alpha$. As usual, the ratio of the scale factor at the emission and absorption of a null ray determines the cosmological redshift z by

$$\frac{a_0}{a(z)} = 1 + z , \tag{3}$$

and the age of the universe is

$$t(z) = \frac{\alpha}{H_0(1+z)^{1/\alpha}} . \tag{4}$$

Using (3), we define the dimensionless Hubble parameter

$$h(z) \equiv \frac{H(z)}{H_0} = (1+z)^{1/\alpha}. \tag{5}$$

The present 'radius' of the universe is defined as (see Eq. 1)

$$a_0 = B\frac{c}{H_0} . \tag{6}$$

In terms of the parameters α and B, the angular diameter distance between two different redshifts is:

$$D_{\mathrm{A}}(z_1, z_2, \alpha) = \frac{Bc}{(1+z_2)H_0}\sinh\left[\frac{1}{B}\frac{\alpha}{\alpha-1}\left\{(1+z_2)^{\frac{\alpha-1}{\alpha}} - (1+z_1)^{\frac{\alpha-1}{\alpha}}\right\}\right] . \tag{7}$$

In a limiting case, $\alpha \to 1$, we obtain the following expression

$$D_{\mathrm{A}}(z_1, z_2) = \frac{Bc}{2H_0}\frac{\left[(1+z_2)^{2/B} - (1+z_1)^{2/B}\right]}{(1+z_1)^{1/B}(1+z_2)^{\frac{B+1}{B}}} . \tag{8}$$

The look-back time, which is the difference between the age of the universe when a particular light ray was emitted and the age of the universe now, we find as

$$\frac{c\,dt}{dz_{\mathrm{L}}} = \frac{c}{H_0(1+z_{\mathrm{L}})^{\frac{\alpha+1}{\alpha}}} . \tag{9}$$

3

3 Testing the model against observations

3.1 Gravitational lensing statistics

We consider a sample of 867 ($z > 1$) high luminosity optical quasars which include 5 lensed quasars (1208 + 1011, H 1413 + 117, LBQS 1009 + 0252, PG 1115 + 080, 0142 + 100). This sample is taken from optical lens surveys such as the HST Snapshot survey [13], the Crampton survey [14], the Yee survey [15], Surdej survey [16], the NOT Survey [17] and the FKS survey [18]. The lens surveys and quasar catalogs usually use V magnitudes, so we transform m_V to a B-band magnitude using an average $B - V$ colour of 0.2 mag [19, 20, 21].

The differential probability $d\tau$ of a beam having a lensing event in traversing $dz_{\rm L}$ is

$$d\tau = n_0(1 + z_L)^3 \, \sigma \, \frac{cdt}{dz_{\rm L}} dz_{\rm L} \, , \tag{10}$$

where n_0 is the present comoving number density of the lenses, σ is the cross-section for lensing event and the quantity $cdt/dz_{\rm L}$ is given by (9).

For simplicity we use the Singular Isothermal Sphere (SIS) model for the lens mass distribution. The cross-section for lensing events for the SIS model is given by [22]

$$\sigma = 16 \, \pi^3 \left(\frac{v}{c} \right)^4 \left(\frac{D_{\rm OL} D_{\rm LS}}{D_{\rm OS}} \right)^2 \, , \tag{11}$$

where v is the velocity dispersion of the dark halo of the lensing galaxy. We define $D_{\rm OL}$, $D_{\rm OS}$ and $D_{\rm LS}$ as the angular diameter distances from the observer to the lens, to the source and between the lens and the source, respectively.

Assuming no evolution of the galaxies, the comoving number density is modeled by Schechter function as

$$\Phi(L, z = 0) \, dL = \phi_* \, \left(\frac{L}{L_*} \right)^{\tilde{\alpha}} \, \exp \left(-\frac{L}{L_*} \right) \frac{dL}{L_*} \, , \tag{12}$$

where ϕ_*, $\tilde{\alpha}$ and L_* are the normalization factor, the index of faint-end slope, and the characteristic luminosity, respectively. We also assume that the velocity dispersion of dark matter halo v is related to the luminosity L by the Faber-Jackson relation for E/S0 galaxies $v = v_* \, (L/L_*)^{1/\gamma}$.

The differential probability is given by[23]

$$d\tau = F^*(1 + z_{\rm L})^3 \frac{H_0}{c} \left(\frac{H_0 D_{\rm OL} D_{\rm LS}}{c \, D_{\rm OS}} \right)^2 \frac{cdt}{dz_{\rm L}} dz_{\rm L} \, , \tag{13}$$

where

$$F^* = \frac{16\pi^3}{cH_0^3} \phi_* v_*^4 \Gamma \left(\tilde{\alpha} + \frac{4}{\gamma} + 1 \right) \tag{14}$$

is the dimensionless quantity, which measures the effectiveness of matter in producing multiple images. Table 1 lists Schechter and lens parameters for E/S0 galaxies, as suggested by

[24] (hereafter *LPEM* parameters). We neglect the contribution of spirals as lenses, as their velocity dispersion is small in comparison to E/S0 galaxies.

The constraints obtained on the cosmological paramters are highly dependent on the choice of lens and Schechter paramters. The lens and Schechter parameters should be determined in a highly correlated manner from a galaxy survey. The use of parameters derived from various surveys might introduce error. We consider LPEM parameters as they form one such set of paramters and they also take into account the morphological distribution of the E/S0 galaxies[25]. Recently several galaxy surveys have come up with a much larger sample of galaxies. This has improved our knowledge of the galaxy luminosity function. However, these surveys donot classify the galaxies by their morphological type [26, 27, 28].

The differential optical depth of lensing in traversing dz_L with angular separation between ϕ and $\phi + d\phi$ reads as

$$\frac{d^2\tau}{dz_L d\phi} d\phi dz_L =$$

$$F^*(1+z_L)^3 \frac{H_0}{c} \frac{\gamma/2}{\Gamma\left(\tilde{\alpha}+1+\frac{4}{\gamma}\right)} \frac{cdt}{dz_L} \left(\frac{H_0 D_L D_{LS}}{c\, D_S}\right)^2 \left(\frac{D_S}{D_{LS}}\phi\right)^{\frac{\gamma}{2}\left(\tilde{\alpha}+1+\frac{4}{\gamma}\right)} \exp\left[\left(-\frac{D_S}{D_{LS}}\phi\right)^{\frac{\gamma}{2}}\right] \frac{d\phi}{\phi} dz_L \,, \tag{15}$$

where $\phi = \Delta\theta/8\pi(v_*/c)^2$ and v_* the velocity dispersion corresponding to the characteristic luminosity L_* in (12).

We make two corrections to the optical depth to get the lensing probability: magnification bias and selection function. Magnification bias, $\mathbf{B}(m, z)$, is to take into account the increase in the apparent brightness of a quasar due to lensing, which, in turn, increases the expected number of lenses in flux limited sample.

The bias factor for a quasar at redshift z with apparent magnitude m is given by [29, 30, 31]

$$\mathbf{B}(m, z) = M_0^2\, B(m, z, M_0, M_2) \,, \tag{16}$$

where

$$B(m, z, M_1, M_2) = 2\left(\frac{dN_Q}{dm}\right)^{-1} \int_{M_1}^{M_2} \frac{dM}{M^3} \frac{dN_Q}{dm}\left(m + 2.5\log(M), z\right). \tag{17}$$

In the above equation $(dN_Q(m, z)/dm)$ is the measure of number of quasars with magnitudes in the interval $(m, m+dm)$ at redshift z. We can allow the upper magnification cutoff M_2 to be infinite, though in practice we set it to be $M_2 = 10^4$. M_0 is the minimum magnification of a multiply imaged source and for the SIS model $M_0 = 2$.

We use Kochanek's "best model" (K96) for the quasar luminosity function:

$$\frac{dN_Q}{dm}(m, z) \propto \left(10^{-a(m-\overline{m})} + 10^{-b(m-\overline{m})}\right)^{-1} \,, \tag{18}$$

where the bright-end slope a and faint-end slope b are constants, and the break magnitude

5

$\overline{m}$ evolves with redshift:

$$\overline{m} = \begin{cases} m_o + (z-1) & \text{for } z < 1, \\ m_o & \text{for } 1 < z \le 3, \\ m_o - 0.7(z-3) & \text{for } z > 3. \end{cases} \tag{19}$$

Fitting this model to the quasar luminosity function data in [32] for $z > 1$, Kochanek finds that "the best model" has $a = 1.07 \pm 0.07$, $b = 0.27 \pm 0.07$ and $m_o = 18.92 \pm 0.16$ at B magnitude. The magnitude corrected probability, p_i, for the quasar i with apparent magnitude m_i and redshift z_i to get lensed is:

$$p_i = \tau(z_i)\mathbf{B}(m_i, z_i) . \tag{20}$$

Selection effects are caused by limitations on dynamic range, limitations on resolution and presence of confusing sources such as stars. The survey can only detect lenses with magnifications larger than M_f. This sets the lower limit on the magnification. Therefore the M_1 in the bias function (17) gets replaced by $M_f(\theta)$ (for details, see K93), which is given as

$$M_f = M_0(f + 1)/(f - 1) , \tag{21}$$

with

$$f = 10^{0.4\,\Delta m(\theta)} . \tag{22}$$

The corrected lensing probability and image separation distribution function for a single source at redshift z_{S} are given in K96

$$p_i'(m, z) = p_i \int \frac{d(\Delta\theta)\, p_c(\Delta\theta) B(m, z, M_f(\Delta\theta), M_2)}{B(m, z, M_0, M_2)}, \tag{23}$$

and

$$p_{ci}' = p_{ci}(\Delta\theta) \frac{p_i}{p_i'} \frac{B(m, z, M_f(\Delta\theta), M_2)}{B(m, z, M_0, M_2)} , \tag{24}$$

where

$$p_c(\Delta\theta) = \frac{1}{\tau(z_S)} \int_0^{z_S} \frac{d^2\tau}{dz_L d(\Delta\theta)} \, dz_L . \tag{25}$$

Equation (24) defines the configuration probability. It is the probability that the lensed quasar i is lensed with the observed image separation.

To get selection function corrected probabilities, we divide our sample into two parts—the ground based surveys and the HST survey. We use the selection functions as suggested in K93.

In our present calculations we do not consider the extinction effects due to the presence of dust in the lensing galaxies.

Finally, we use the above basic equations to perform the following tests :

6

(i) The sum of the lensing probabilities $p_i^{'}$ for the optical QSOs gives the expected number of lensed quasars, $n_{\rm L} = \sum p_i^{'}$. The summation is over the given quasar sample. We look for those values of the parameter for which the adopted optical sample has exactly five lensed quasars (that is those values of the parameters for which $n_{\rm L} = 5$).

We start with a two parameter fit. We allow α to vary in the range $(0.0 \leq \alpha \leq 2.0)$ and B to vary in the range $(0.5 \leq B \leq 10.0)$. We observe that for $B \geq 1$, D_A becomes independent of it. It can be easily checked from equation (7) that for large values of B, $\sinh(x) \sim x$ making D_A independent of B. From the previous works constraining power law cosmology [11, 12], the value of $B = 1$ is found to be compatible with observations. Incidentally, we can estimate the present scale factor of the universe as $a_0 \approx c/H_0$, hence we use $B = 1$ in further analysis.

Fig. 1 shows the predicted number of lensed quasars for the above specified range of α. We obtain $n_{\rm L} = 5$ for $\alpha = 1.06$. We further generate 10^4 quasar samples (each sample has 867 quasars) using bootstrap method and find best fit α for each data set to obtain error bars on α. We finally obtain $\alpha = 1.09 \pm 0.3$.

(ii) We also perform maximum likelihood analysis to determine the value of α, for which the observed sample becomes the most probable observation. The likelihood function is

$$\mathcal{L} = \prod_{i=1}^{N_{\rm U}}(1 - p_i^{'}) \prod_{k=1}^{N_{\rm L}} p_k^{'} p_{ck}^{'} . \tag{26}$$

Here $N_{\rm L}$ is the number of multiple-imaged lensed quasars, $N_{\rm U}$ is the number of unlensed quasars, $p_k^{'}$, the probability of quasar k to get lensed is given by Eq. 23 and p_{ck}^{i}, the configuration probability, is given by Eq. 24. The best fit ($\mathcal{L}_{\rm max}$) occurs for $\alpha = 1.13$. We see that $0.85 \leq \alpha \leq 1.56$ at 1σ (68% confidence level) and $0.65 \leq \alpha \leq 2.33$ at 2σ (95.4% confidence level).

However, gravitational lensing statistics is susceptible to a number of uncertainties. The constraints obtained on the cosmological parameters from the statistics of strong lensing may vary after inclusion of the uncertainties in the luminosity function, lensing cross-section for galaxies (E/S0) and quasar luminosity function, role of spirals and the dust extinction.

3.2 Constraints from age estimates of high-z galaxies

Another observational test which can constraint α is the age measurement of the old high redshift galaxies (OHRG)[33, 34]. These constraints are more stringent than those obtained from globular cluster age measurements [35, 2, 36]. Here, we consider the galaxy 3C65 at $z = 1.175$ (4 Gyr old) [37], at $z = 1.55$ (3.5 Gyr old; 53W091) [35, 38] and a 4 Gyr old galaxy 53W069 at $z = 1.43$ [39]. These are the minimum ages of these galaxies as indicated by best fitting spectral synthesis models. The age of the universe at a given redshift is greater than or at least equal to the age of its oldest objects at that redshift. In power law cosmologies the age of the universe increases with increasing α. Hence this test gives lower bound on α.

This can be checked if we define the dimensionless ratio:

$$\frac{t(z)}{t_{\mathrm{g}}} = \frac{f(\alpha, z)}{H_0 t_{\mathrm{g}}} \geq 1 \; . \tag{27}$$

The t_{g} is the age of an old object and $f(\alpha, z) = \alpha/(1+z)^{1/\alpha}$. The error bar on H_0 determines the extreme value of t_{g}. The lower limit on H_0 was recently updated to nearly 10% of accuracy by Freedman [40]: $H_0 = 70\pm7$ km/sec/Mpc; 1σ. The constraints from SNe Ia data also point to $H_0 > 60$ km/sec/Mpc [41]. For the galaxy 3C65, the lower limit on age (4.0 Gyr) yields: $0.26 \leq H_0\, t_{\mathrm{g}} \leq 0.32$. Similarly for the galaxy 53W069 we have $0.26 \leq H_0\, t_{\mathrm{g}} \leq 0.32$. For the galaxy 53W091 at $z = 1.55$ the lower limit on age (3.5 Gyr) gives $0.23 \leq H_0\, t_{\mathrm{g}} \leq 0.28$.

Fig. 2 shows the variation of the function $f(\alpha, z)$ with the redshift z for several values of α. We see that α should be at least 0.8, in order to allow for these OHRG to exist.

4   Summary and a combined constraint

The main results of the present paper along the with constraints obtained from the SNe Ia data [11] are summarized in Table 2. The motivation for our work was to establish the viability of a linear coasting cosmology $a(t) = t$. Using gravitational lensing statistics, we find that such a coasting is accommodated within 1σ. The age determination of OHRG gives as a lower bound $\alpha \geq 0.8$ for a power law cosmology $a(t) \propto t^{\alpha}$. Moreover, for such a cosmology $H_0 t_0 = \alpha$. With updated value of $H_0 = 70\pm7$ km/sec/Mpc and $t_0 = 14\pm2$ Gyr [42], it gives $\alpha = 0.98\pm0.25$. Dev et al (2001) reported that $\alpha = 1.0$ is consistent with SNe Ia data (within 68% confidence level). We find that $\alpha = 1.0$ is in concordance with the listed observational tests. It is interesting to observe that the lensing analysis barely accomodates an Einstein-de Sitter universe ($\alpha = 2/3$) at 2σ. Similarly, the age determination of OHRG also rules out $\alpha = 2/3$. We conclude that the coasting cosmology with strictly linear evolution of scale factor, $a(t) = t$, cannot be ruled out on the basis of these observations.

Acknowledgments

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Survey	$\tilde{\alpha}$	γ	$v^*(Km/s)$	$\phi^*(Mpc^{-3})$	F^*
LPEM	+0.2	4.0	205.3	$3.2 \pm 0.17h^3 \times 10^{-3}$	0.010

Table 1: Lens and Schechter parameters for E/S0 galaxies.

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Method	Reference	α
Lensing Statistics		
(i) n_{L}	This paper	1.09 ± 0.3
(ii) Likelihood Analysis	This paper	$1.13^{+0.4}_{-0.3}$
OHRG	This paper	≥ 0.8
SNe Ia	Dev et al.(2001)	1.004 ± 0.043

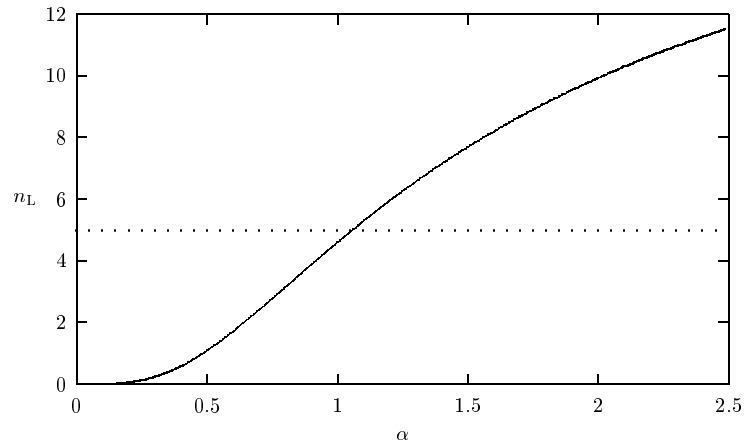Table 2: Constraints on α from various cosmological tests.

Figure 1: Predicted number of lensed quasars n_{L} in the adopted optical quasar sample, with image separation $\Delta\theta \leq 4$, Vs power index α.
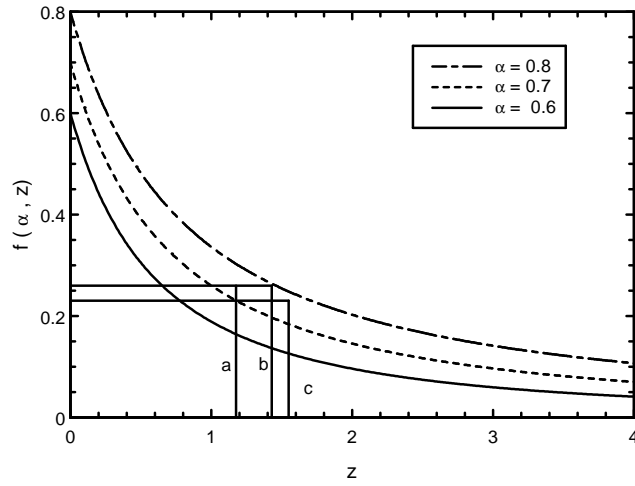
Figure 2: $f(\alpha, z)$ vs. z for various different values of α; 'a' corresponds 3C65 galaxy ($z = 1.175$), 'b' corresponds to the galaxy 53W069 ($z = 1.43$) and 'c' corresponds to 53W091 ($z = 1.55$).